Analytic and experimental results of spatial correlations of vector intensity sensors

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I. INTRODUCTION

In array applications of acoustic sensors, the effects of ambient noise on the correlation between various sensor pairs should be understood to maximize array performance. While idealized, a homogenous, isotropic noise field is generally assumed in order to derive analytic solutions for the correlations of spatially separated sensors. Spatial correlations for two separated pressure sensors were investigated as early as 1955 [1]. This and other studies [2], [3] have led to determinations of optimal pressure sensor separations to suppress the ambient noise in the acoustic field.

Over the past decade, demonstrated benefits in noise rejection and signal extraction of intensity vector sensors has spurred interest in spatial arrays of acoustic vector sensors [4]-[6]. However, to date no work has been published detailing spatial correlations of acoustic intensity components. This paper extends the classical solutions to derive analytic forms for the correlations of spatially separated intensity field components.

II. ANALYTIC SOLUTIONS

Several techniques have been employed to derive the classical sin(*kd*)/(*kd*) solution for the spatial correlation of the isotropic pressure field, where k is the acoustic wave number and d is the separation distance between the locations in the field. These analytic techniques have also been expanded to include spatial correlations between acoustic pressure, p, and velocity, u [7]-[8].

The intensity correlations solutions proposed here are determined for three cases: 1) intensity components parallel to the separation vector, **d**; 2) components orthogonal to **d** but parallel to each other; 3) orthogonal intensity components. To simplify the derivation a Gaussian distribution of the sound field is assumed [9]. Closed form, normalized spatial correlations, $\rho(d)$, for each of the various intensity sensor pairs are shown to be

$$\rho_{II_{\parallel}}(d) = 3\frac{\left((kd)^2 - 2\right)\sin^2(kd) + 2kd\cos(kd)\sin(kd)}{(kd)^4}, \tag{1}$$

$$\rho_{II_{\perp}}(d) = 3\frac{\sin^2(kd) - kd\sin(kd)\cos(kd)}{(kd)^4}, \tag{2}$$

where the subscript indicates the intensity component orientations (the correlation for case 3 is identically zero). These are illustrated in Fig. 1, which includes the classic pressure–pressure correlation for comparison. The intensity correlations converge to zero more rapidly than similar pressure or velocity correlations, indicating that an intensity-processed array of vector sensors may be less susceptible to ambient background noise contamination.

III. EXPERIMENTAL VALIDATION

The derived analytic intensity correlations are verified in two separate experiments. First, the correlations are examined computationally using MATLAB. A random isotropic noise field is modeled as the sum of a large number N of plane waves generated from directions evenly distributed around two sensor locations, approximating an ideal isotropic sound field. The auto- and cross-spectra and the coherence between the sensor signals are determined. The coherence measures between the two sensors are compared to the squared envelope of the normalized correlation solutions given in (1) and (2) for the appropriate

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combination of sensor types [8]. Results corroborate the derivation methods for correlations both of individual p and u components and of intensity-processed vector sensors. The measured intensity coherences are plotted in Fig. 2 along with corresponding values for the squared, normalized analytical expression.

A second verification of the derived intensity correlations is completed in underwater experiments performed in an acoustic tank facility at the Pennsylvania State University. A pair of inertial vector sensors developed by McConnell, Weber, Lauchle and Gabrielson [10] is placed in a large reverberant tank, which is excited acoustically by two independent sources generating continuous broadband white noise. Measurements of acoustic pressure and two particle velocity components of both sensors are recorded to a digital storage media. Using the same analysis tool employed above, the measured coherences are also plotted in Fig. 2, along with the computational results and the theoretical expressions. Again, the data clearly corroborate the derived analytic solutions for spatial correlations of intensity-processed vector sensors.

IV. CONCLUSIONS

The derived theoretical expressions for spatially separated intensity measures have been extensively verified in two separate experiments. Computational methods corroborate the derived solutions, as do the physical measurements using spatially separated acoustic vector sensors in an isotropic sound field. The measured results show excellent agreement with the computational model. Further, due to the inherent "noiseless" nature of the computational experimental model, the effects of other issues on the various coherences can also be easily determined. From this study, a better understanding of the effects of ambient noise on multiply processed pressure, velocity, and intensity sensors is gained. It is hoped that these findings will help direct future design efforts for vector sensor arrays.

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REFERENCES

[1] R. K. Cook, R. V. Waterhouse, R. D. Berendt, S. Edelman, and M. C. Thompson, "Measurement of correlation coefficients in reverberant sound fields," *J. Acoust. Soc. Am., 27*, 1071–1077.

[2] F. Jacobsen and T. G. Nielsen, "Spatial correlation and coherence in a reverberant sound field," *J. Sound Vib., 118*, 175–180.

[3] R. V. Waterhouse, "Statistical properties of reverberant sound fields," *J. Acoust. Soc. Am., 43*, 1436–1444.

[4] G. C. Lauchle, "Effect of turbulent boundary layer flow on measurement of acoustic pressure and intensity," *Noise Control Eng. J., 23*, 52–59.

[5] B. R. Rapids and G. C. Lauchle, "Vector intensity field scattered by a rigid prolate spheroid," *J. Acoust. Soc. Am., 120*, 38–48.

[6] N. K. Naluai, G. C. Lauchle, T. B. Gabrielson, J. H. Joseph, "Bi-static Sonar Applications of Intensity Processing," *J. Acoust. Soc. Am., 121*, 1909-1915.

[7] G. L. D'Spain, J. C. Luby, G. R. Wilson, and R. A. Gramann, "Vector sensors and vector sensor line arrays: Comments on optimal array gain and detection." *J. Acoust. Soc. Am., 120*, 171–185.

[8] F. Jacobsen and T. Roisin, "Spatial coherence in reverberant sound fields," In *Internoise '99*, Fort Lauderdale, FL, 1999, pp.1559–1564.

[9] J. S. Bendat and A. G. Piersol, *Engineering Applications of Correlation and Spectral Analysis, 3rd Edition*, New York, NY: John Wiley & Sons, Inc., 1993, pp. 64-73.

[10] J. A. McConnell, T. C. Weber, G. C. Lauchle, and T. B. Gabrielson, "Development of a high frequency underwater acoustic intensity probe," In *Proc. Oceans 2002*, Biloxi, MS, 2002, pp. 1924–1929.

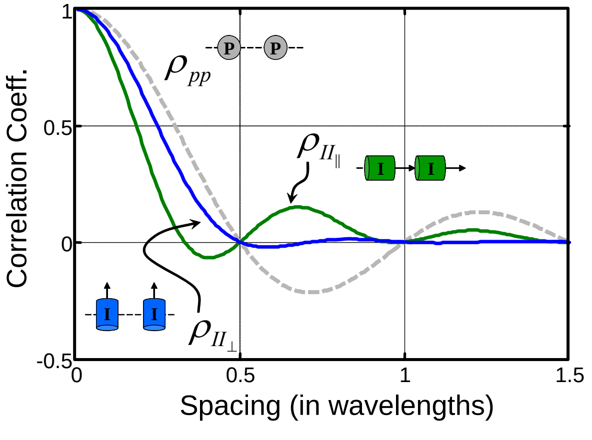

Fig. 1. Illustration of the correlations of two orientations of separated intensity-processed vector sensors versus the sensor separation distance in wavelengths. The intensity correlations share the classic cyclic form of the pressure correlation, but show a dramatic decrease in magnitude with increasing frequency.

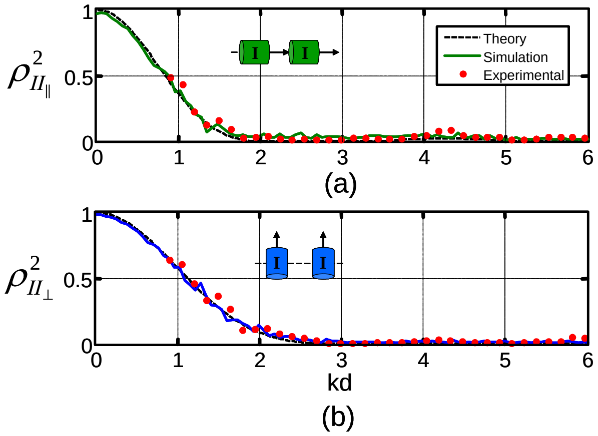

Fig. 2. Comparison of the theoretical intensity correlation solutions to measured results from computational and physical experiments versus *kd*. Both data sets validate the derived analytic solutions, showing excellent agreement at all frequencies. Experimental data is truncated below ~1-kHz due to severe low-frequency rolloff of the acoustic sources.